

# Calculations in unit groups of modular group algebras using the GAP4 package LAGUNA 3.0

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LAGUNA 3.0 is a software package which was developed by the authors to extend the computer algebra system GAP (<http://www.gap-system.org>) for computations in group rings. For the LAGUNA 3.0 manual and distribution see its homepage at the address <http://ukrgap.exponenta.ru/laguna.htm>.

Besides computing some general properties and attributes of group rings and their elements, LAGUNA is able to perform two main kinds of computations. Namely, it can verify whether a group algebra of a finite group satisfies certain Lie properties; and it can calculate the structure of the normalized unit group of a group algebra of a finite  $p$ -group over the field of  $p$  elements.

Let  $FG$  be a group algebra of a finite  $p$ -group  $G$  over the field  $F$  of  $p$  elements. A unit of  $FG$  of the form  $\alpha_1 g_1 + \alpha_2 g_2 + \dots + \alpha_k g_k$  with  $\alpha_i \in F$  and  $g_i \in G$  is said to be normalised if  $\alpha_1 + \alpha_2 + \dots + \alpha_k = 1$ . It is easy to see that the unit group of  $FG$  is the direct product of  $F^* = F \setminus \{0\}$  and the group of normalised units  $V(FG)$ .

The normalised unit group  $V(FG)$  is a finite  $p$ -group of order  $|F|^{|G|-1}$ . For efficient computations inside this group, in the LAGUNA package it is possible to compute its power-commutator presentation, given by generators  $y_1, \dots, y_{|G|-1}$  and two types of relations:

$$y_i^p = (y_{i+1})^{\alpha_{i,i+1}} \dots (y_{|G|-1})^{\alpha_{i,|G|-1}}$$

for  $1 \leq i \leq |G| - 1$ , and

$$[y_j, y_i] = (y_{j+1})^{\alpha_{j,i,j+1}} \dots (y_{|G|-1})^{\alpha_{j,i,|G|-1}}$$

for  $1 \leq i < j \leq |G| - 1$ , where the exponents  $\alpha_{i,k}$  and  $\alpha_{i,j,k}$  are elements of the set  $\{0, \dots, p-1\}$ . Having constructed such a presentation, we can successfully use all methods provided by GAP for such groups, and then map our results back to the group algebra to get their ring-theoretic interpretation. Thus, LAGUNA 3.0 gives new opportunities for computer-aided investigations of some problems existing in the theory of group rings.

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