# Unified Approach to Fourier–Clifford–Prometheus Sequences, Transforms and Filter Banks

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Discrete classical Fourier–Prometheus Transforms (FPT) in bases of different Golay–Rudin–Shapiro sequences can be used in many signal processing applications: digital audition, digital video broadcasting, communication systems (Orthogonal Frequency Division Multiplexing, Multi–Code Code–Division Multiple Access) radar and cryptographic systems.

Golay–Rudin–Shapiro 2–complementary sequences associated with the cyclic group $\mathbf{Z}_2$ were introduced by Golay and Shapiro in 1949–1951. In 1961, Golay gave an explicit construction for binary Golay complementary pairs of length $2^m$. Budisin gave a more general recursive construction for Golay complementary pairs. Recently, Davis and Jedwab gave an explicit description of a large class of Golay complementary sequences in terms of certain cosets of the first order Reed–Muller codes. It is known that general building elements of classical Fourier–Prometheus transforms in bases of classical Golay–Rudin–Shapiro and Davis–Jedwab sequences are a dyadic Abelian group $\mathbf{Z}_2^n$, 2–point Fourier transform $\mathcal{F}_2$ and the complex field $\mathbf{C}$, i.e. these transforms are associated with the triple $(\mathbf{Z}_2^n, \mathcal{F}_2, \mathbf{C})$.

In this paper we develop a new unified approach to generalized so–called *Fourier–Clifford–Prometheus sequences* and *transforms*. It based on a new generalized FCPT–generating construction. This construction has a rich and very pleasing algebraical structure that supports a wide range of fast, elegant, and practical algorithms. It is associated not with the triple $\langle\langle \mathbf{Z}_2^n, \mathcal{F}_2, \mathbf{C}\rangle\rangle$ but rather with other groups (Abelian groups, orthogonal and unitary groups $\mathbf{SO}(2)$–, $\mathbf{SO}(n)$–,$\mathbf{SU}(2)$–, and $\mathbf{SU}(n)$), other fields, rings and algebras (triplet color algebra, multiplet multicolor algebra, hypercomplex commutative algebras, Clifford algebras). Further, this construction inherits some useful algebraic properties from the classical FPT–construction and allows a large class of discrete orthogonal and unitary Fourier–Clifford transforms to be generated from a single recursion formula. Finally, properties and characteristics of Fourier–Clifford–Prometheus sequences and transforms are derived in the group–theoretical and algebraic framework.