GROUP FILTERS AND IMAGE PROCESSING

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Abstract Abelian group DSP can be completely described in terms of a special class of signals, the characters, defined by their relationship to the translations defined by abelian group multiplication. The first problem to be faced in extending classical DSP theory is to decide on what is meant by a translation. We have selected certain classes of nonabelian groups and defined translations in terms of left nonabelian group multiplications.

The main distinction between abelian and nonabelian group DSP centers around the problem of character extensions. For abelian groups the solution of the character extension problem is simple. Every character of a subgroup of an abelian group A extends to a character of A. We will see that character extensions lie at the heart of several fast Fourier transform (FFT) algorithms.

The nonabelian groups presented in this work will be among the simplest generalizations of abelian groups. A complete description of the DSP of an abelian by abelian semidirect product will be given. For such a group G, there exists an abelian normal subgroup A. A basis for signals indexed by G can be constructed by extending the characters of A and then by forming certain (left-) translations of these characters. The crucial point is that some of these characters of A cannot be extended to characters of G, but only to proper subgroups of G. In contrast to abelian group DSP expansions, expansions in nonabelian group DSP reflect local signal domain information over these proper subgroups. These expansions can be viewed as combined, local image-spectral image domain expansions in analogy to time-frequency expansions.

This DSP leads to the definition of a certain class of group filters whose properties will be explored on simulated and recorded images. Through examples we will compare nonabelian group filters with classical abelian group filters. The main observation is that in contrast to abelian group filters, nonabelian group filters can localize in the image domain. This advantage has been used for detecting and localizing the position of geometric objects in image data sets.

The works of R. Holmes M. Karpovski and E. Trachtenberg are the basis of much of the research in the application of nonabelian group harmonic analysis to DSP, especially in the construction of group transforms and group filters. Similar ideas in coding theory have been introduced by F. J. MacWilliams.

During the last ten years considerable effort has taken place to extend the range of applicability of nonabelian group methods to the design of new filters and spectral analysis methodologies, as an image processing tool. Accompanying these application efforts, there has been extensive development of algorithms for computing Fourier transforms over nonabelian groups generalizing the classical FFT.

Keywords: finite group harmonic analysis, group filters, image domain locality.