CLIFFORD GEOMETRIC ALGEBRAS IN MULTILINEAR ALGEBRA AND NON-EUCLIDEAN GEOMETRIES

Garret Sobczyk*

Universidad de las Américas
Departamento de Físico-Matemáticas
Apartado Postal #100, Santa Catarina Mártir
72820 Cholula, Pue., México

sobczyk@mail.udlap.mx

Abstract Given a quadratic form on a vector space, the geometric algebra of the corresponding pseudo-euclidean space is defined in terms of a simple set of rules which characterizes the geometric product of vectors. We develop geometric algebra in such a way that it augments, but remains fully compatible with, the more traditional tools of matrix algebra. Indeed, matrix multiplication arises naturally from the geometric multiplication of vectors by introducing a spectral basis of mutually annihiliating idempotents in the geometric algebra. With the help of a few more algebraic identities, and given the proper geometric interpretation, the geometric algebra can be applied to the study of affine, projective, conformal and other geometries. The advantage of geometric algebra is that it provides a single algebraic framework with a comprehensive, but flexible, geometric interpretation. For example, the affine plane of rays is obtained from the euclidean plane of points by adding a single anti-commuting vector to the underlying vector space. The key to the study of noneuclidean geometries is the definition of the operations of meet and join, in terms of which incidence relationships are expressed. The horosphere provides a homogeneous model of euclidean space, and is obtained by adding a second anti-commuting vector to the underlying vector space of the affine plane. Linear orthogonal transformations on the higher dimensional vector space correspond to conformal or Möbius transformations on the horosphere. The horosphere was first constructed by F.A. Wachter (1792–1817), but has only recently attracted attention by offering a host of new computational tools

*I gratefully acknowledge the support given by INIP of the Universidad de las Américas. The author is a member of SNI, Exp. 14587.

in projective and hyperbolic geometries when formulated in terms of geometric algebra.

Keywords: affine geometry, Clifford algebra, conformal geometry, conformal group, euclidean geometry, geometric algebra, horosphere, Möbius transformation, non-euclidean geometry, projective geometry, spectral decomposition.