Calculations in unit groups of modular group algebras using the GAP4 package LAGUNA 3.0

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LAGUNA 3.0 is a software package which was developed by the authors to extend the computer algebra system GAP (http://www.gap-system.org) for computations in group rings. For the LAGUNA 3.0 manual and distribution see its homepage at the address http://ukrgap.exponenta.ru/laguna.htm.

Besides computing some general properties and attributes of group rings and their elements, LAGUNA is able to perform two main kinds of computations. Namely, it can verify whether a group algebra of a finite group satisfies certain Lie properties; and it can calculate the structure of the normalized unit group of a group algebra of a finite p-group over the field of p elements.

Let FG be a group algebra of a finite p-group G over the field F of p elements. A unit of FG of the form $\alpha_1 g_1 + \alpha_2 g_2 + \cdots + \alpha_k g_k$ with $\alpha_i \in F$ and $g_i \in G$ is said to be normalised if $\alpha_1 + \alpha_2 + \cdots + \alpha_k = 1$. It is easy to see that the unit group of FG is the direct product of $F^* = F \setminus \{0\}$ and the group of normalised units $V(FG)$.

The normalised unit group $V(FG)$ is a finite p-group of order $|F|^{|G|-1}$. For efficient computations inside this group, in the LAGUNA package it is possible to compute its power-commutator presentation, given by generators $y_1, \ldots, y_{|G|-1}$ and two types of relations:

$$y_i^p = (y_{i+1})^{\alpha_{i,i+1}} \cdots (y_{|G|-1})^{\alpha_{i,|G|-1}}$$

for $1 \leq i \leq |G| - 1$, and

$$[y_j, y_i] = (y_{j+1})^{\alpha_{j,i,j+1}} \cdots (y_{|G|-1})^{\alpha_{j,i,|G|-1}}$$

for $1 \leq i < j \leq |G| - 1$, where the exponents $\alpha_{i,k}$ and $\alpha_{i,j,k}$ are elements of the set $\{0, \ldots, p-1\}$. Having constructed such a presentation, we can successfully use all methods provided by GAP for such groups, and then map our results back to the group algebra to get their ring-theoretic interpretation. Thus, LAGUNA 3.0 gives new opportunities for computer-aided investigations of some problems existing in the theory of group rings.

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